

## Problem 2.13

[Difficulty: 3]

**2.13** A flow field is given by

$$\vec{V} = -\frac{qx}{2\pi(x^2 + y^2)}\hat{i} - \frac{qy}{2\pi(x^2 + y^2)}\hat{j}$$

where  $q = 5 \times 10^4 \text{ m}^2/\text{s}$ . Plot the velocity magnitude along the  $x$  axis, along the  $y$  axis, and along the line  $y = x$ , and discuss the velocity direction with respect to these three axes. For each plot use a range  $x$  or  $y = -1 \text{ km}$  to  $1 \text{ km}$ , excluding  $|x|$  or  $|y| < 100 \text{ m}$ . Find the equation for the streamlines and sketch several of them. What does this flow field model?

**Given:** Flow field

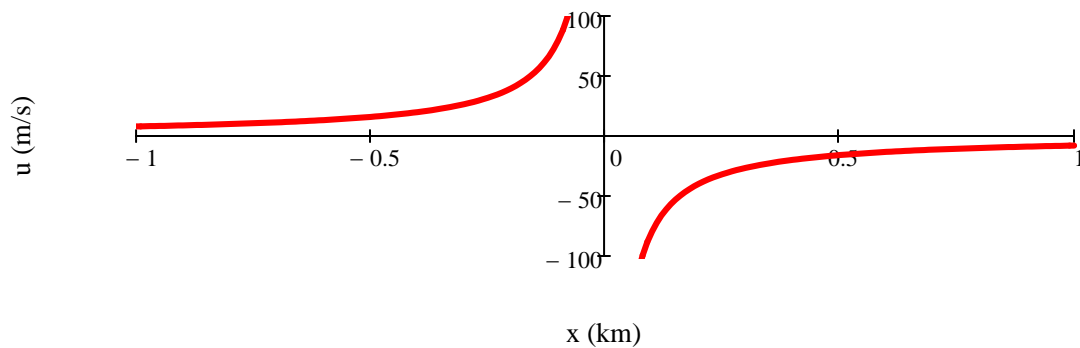
**Find:** Plot of velocity magnitude along axes, and  $y = x$ ; Equations of streamlines

**Solution:**

On the  $x$  axis,  $y = 0$ , so

$$u = -\frac{q \cdot x}{2 \cdot \pi \cdot (x^2 + y^2)} = -\frac{q}{2 \cdot \pi \cdot x} \quad v = -\frac{q \cdot y}{2 \cdot \pi \cdot (x^2 + y^2)} = 0$$

Plotting

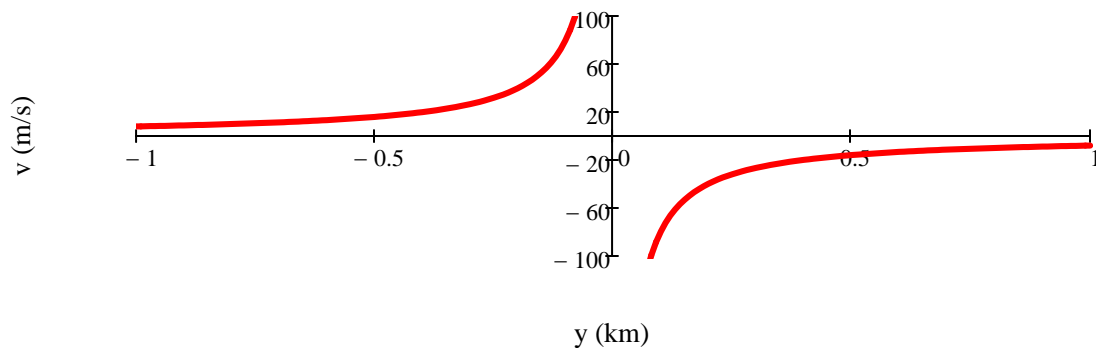


The velocity is very high close to the origin, and falls off to zero. It is also along the axis. This can be plotted in *Excel*.

On the  $y$  axis,  $x = 0$ , so

$$u = -\frac{q \cdot x}{2 \cdot \pi \cdot (x^2 + y^2)} = 0 \quad v = -\frac{q \cdot y}{2 \cdot \pi \cdot (x^2 + y^2)} = -\frac{q}{2 \cdot \pi \cdot y}$$

Plotting



The velocity is again very high close to the origin, and falls off to zero. It is also along the axis.

This can also be plotted in *Excel*.

On the  $y = x$  axis

$$u = -\frac{q \cdot x}{2 \cdot \pi \cdot (x^2 + x^2)} = -\frac{q}{4 \cdot \pi \cdot x} \quad v = -\frac{q \cdot x}{2 \cdot \pi \cdot (x^2 + x^2)} = -\frac{q}{4 \cdot \pi \cdot x}$$

The flow is parallel to line  $y = x$ :

Slope of line  $y = x$ : 1

Slope of trajectory of motion:  $\frac{v}{u} = 1$

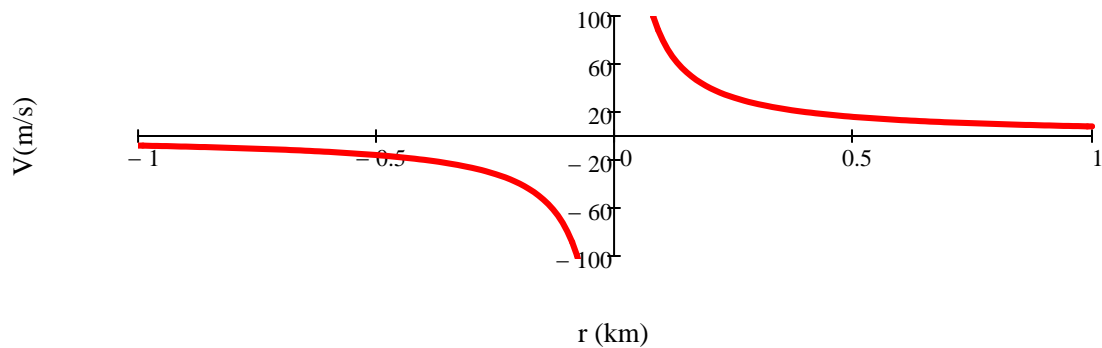
If we define the radial position:

$$r = \sqrt{x^2 + y^2} \quad \text{then along } y = x \quad r = \sqrt{x^2 + x^2} = \sqrt{2} \cdot x$$

Then the magnitude of the velocity along  $y = x$  is

$$V = \sqrt{u^2 + v^2} = \frac{q}{4 \cdot \pi} \cdot \sqrt{\frac{1}{x^2} + \frac{1}{x^2}} = \frac{q}{2 \cdot \pi \cdot \sqrt{2} \cdot x} = \frac{q}{2 \cdot \pi \cdot r}$$

Plotting



This can also be plotted in Excel.

For streamlines

$$\frac{v}{u} = \frac{dy}{dx} = \frac{-\frac{q \cdot y}{2 \cdot \pi \cdot (x^2 + y^2)}}{-\frac{q \cdot x}{2 \cdot \pi \cdot (x^2 + y^2)}} = \frac{y}{x}$$

So, separating variables

$$\frac{dy}{y} = \frac{dx}{x}$$

Integrating

$$\ln(y) = \ln(x) + c$$

The solution is

$$y = C \cdot x \quad \text{which is the equation of a straight line.}$$

This flow field corresponds to a sink (discussed in Chapter 6).